

# Composite Pairings in Chirally Stabilized Critical Fluids

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We study a one-dimensional electron gas in a special antiferromagnetic environment made by two spin-1/2 Heisenberg chains and the one-dimensional two-channel Kondo-Heisenberg lattice away from half-filling. These models flow to an intermediate fixed point which belongs to the universality class of chirally stabilized liquids. Using a Toulouse point approach, the universal properties of the models are determined as well as the identification of the leading instabilities. It is shown that these models exhibit a non-Fermi liquid behavior with strong enhanced composite pairing correlations.

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The concept of odd-frequency superconducting ordering was conceived by Berezinskii [1] who considered spin-triplet odd-frequency pairing as an alternate of the conventional theory <sup>3</sup>He pairing. The possibility of such unusual pairing was revived nearly one decade ago by Balatsky and Abrahams [2] in the context of singlet superconductors and by Emery and Kivelson [3] in the two-channel Kondo problem with the emphasis laid on the composite nature of the order parameter for odd-frequency pairing. This idea of composite pairing was further developed by Coleman et al. [4] who proposed that heavy-fermion superconductors might involve odd-frequency triplet pairing.

One of the most important difficulty in finding some specific lattice models for realization of odd-frequency pairing stems from the fact that it requires a controlled analysis in the strong coupling regime. Such an approach is possible in some extreme limits as in one dimension where non-perturbative techniques are available. Strong odd-frequency singlet pair correlations were, in particular, identified within the bosonization approach [5] of the one-dimensional single channel Kondo lattice in an anisotropic limit [6,7]. Another candidate is the one dimensional single channel Kondo-Heisenberg lattice (KHL) which consists of a one dimensional electron gas (1DEG) interacting with an antiferromagnetic Heisenberg spin-1/2 chain by a Kondo coupling. Away from half filling, this model has a spin gap [8] and is thus expected to exhibit enhanced pairing correlations. In particular, it has been shown recently [9] that the dominant instabilities are of a composite nature in a certain regime of the parameters. In this letter, we shall study two different generalization of the one dimensional KHL with two channels which have no spin gap *but* exhibit dominant unconventional pairing instabilities. A first generalization is to consider a 1DEG coupled by a Kondo coupling to two non-interacting antiferromagnetic Heisenberg spin-1/2 chains. This model belongs to the more general class of Luttinger liquids in active environments introduced in the context of the striped physics [10–12]. A second generalization consists of conduction electrons

with a two-fold orbital degeneracy interacting with a periodic array of localized spins i.e. the two-channel KHL. It has been shown in Ref. [13] that these two models away from half filling exhibit a critical phase governed by a fixed point which belongs to the class of chirally stabilized fluids [14]. In this work, we shall compute correlation functions of physical observables by means of a Toulouse point solution and characterize the dominant instabilities of both models which turn out to be of a composite nature. Very recently, the one dimensional N-channel KHL has been investigated by Andrei and Orignac [15] using a conformal field theory (CFT) approach. These authors found that the leading instability is of a composite pairing type when  $N \leq 4$  [15]. In this respect, the exact solution at the Toulouse point provides an independent and physically transparent approach for the particular 2-channel KHL.

*The models.* The first model (referred in the following as a 1DEG in a special active environment) consists of a 1DEG away from half filling coupled symmetrically by a Kondo coupling ( $J_K > 0$ ) with two non-interacting antiferromagnetic ( $J_H > 0$ ) spin-1/2 Heisenberg chains:

$$\mathcal{H}_1 = -t \sum_i \left( c_{i\sigma}^\dagger c_{i+1\sigma} + H.c. \right) + J_K \sum_{a=1}^2 \sum_i \mathbf{S}_{ci} \cdot \mathbf{S}_{ai} + J_H \sum_{a=1}^2 \sum_i \mathbf{S}_{ai} \cdot \mathbf{S}_{ai+1}. \quad (1)$$

Here,  $c_{i\sigma}$  is the conduction electron operator at site  $i$  with spin  $\sigma = \uparrow, \downarrow$ ,  $\vec{S}_{ci} = c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} / 2$  ( $\vec{\sigma}$  being the Pauli matrices) stands for the electron spin operator at site  $i$  whereas the localized spin operator at site  $i$  on the chain of index  $a = 1, 2$  is denoted by  $\mathbf{S}_{ai}$ . In the limit  $J_K \ll t, J_H$ , the low energy behavior of the model can be determined using the continuum limit of the electron operator in terms of right and left moving fermion fields:  $c_{i\sigma} \rightarrow R_\sigma e^{ik_F x} + L_\sigma e^{-ik_F x}$ ,  $x = ia_0$  ( $k_F$  being the Fermi momentum and  $a_0$  the lattice spacing). The continuum description of the conduction spin density operator  $\mathbf{S}_c$  can then be expressed in terms of a spin current  $\mathbf{J}_{0R,L}$  belonging to

the  $SU(2)_1$  Kac-Moody (KM) algebra [5] and a bosonic field  $\Phi_c = \Phi_{cL} + \Phi_{cR}$  that accounts for the charge degrees of freedom:  $\mathbf{S}_c \rightarrow \mathbf{J}_{0R} + \mathbf{J}_{0L} + \cos(2k_F x + \sqrt{2\pi}\Phi_c)\mathbf{n}_0$ ,  $\mathbf{n}_0$  being the staggered magnetization. In the same way, the spin densities of the surface chains are represented as (see Refs. [16,5]):  $\mathbf{S}_a(x) = \mathbf{J}_a(x) + (-1)^{x/a_0}\mathbf{n}_a(x)$  where  $\mathbf{J}_a = \mathbf{J}_{aR} + \mathbf{J}_{aL}$  and  $\mathbf{n}_a$  are respectively the uniform and staggered parts of the magnetization. The chiral spin currents  $\mathbf{J}_{aR,L}$  belong also to the  $SU(2)_1$  KM algebra. With this low energy description, the Hamiltonian density of the lattice model (1) for incommensurate filling reads as follows in the continuum limit:

$$\mathcal{H}_1 \simeq \frac{v_F}{2} \left( (\partial_x \Phi_c)^2 + (\partial_x \Theta_c)^2 \right) + \frac{2\pi v_F}{3} (\mathbf{J}_{0R}^2 + \mathbf{J}_{0L}^2) + \frac{2\pi v_H}{3} \sum_{a=1}^2 (\mathbf{J}_{aR}^2 + \mathbf{J}_{aL}^2) + g (\mathbf{J}_{0R} \cdot \mathbf{I}_L + \mathbf{J}_{0L} \cdot \mathbf{I}_R) \quad (2)$$

where we have neglected all oscillatory, marginal irrelevant contributions as well as current-current interactions of the same chirality. As shown in Ref. [13], at the strong coupling fixed point, these latter interactions only lead to renormalization of velocities and logarithmic corrections. In Eq. (2),  $\Theta_c = \Phi_{cL} - \Phi_{cR}$  is the dual charge field,  $v_F$  the Fermi velocity,  $v_H$  the spin velocity of the magnetic environment, and  $g \simeq J_K a_0 > 0$  is the interacting coupling constant;  $\mathbf{I}_{R,L} = \mathbf{J}_{1R,L} + \mathbf{J}_{2R,L}$  is a  $SU(2)_2$  KM current being the sum of two  $SU(2)_1$  KM currents.

The second model considered in this letter is the one-dimensional two-channel KHL:

$$\mathcal{H}_2 = -t \sum_i \sum_{a=1}^2 \left( c_{ia\sigma}^\dagger c_{i+1a\sigma} + H.c. \right) + J_H \sum_i \mathbf{S}_{0i} \cdot \mathbf{S}_{0i+1} + J_K \sum_{a=1}^2 \sum_i \mathbf{S}_{aci} \cdot \mathbf{S}_{0i} \quad (3)$$

where now the electron operator  $c_{ia\sigma}$  has a channel index  $a = 1, 2$  and interacts by a Kondo coupling ( $J_K > 0$ ) with a periodic array of localized spins  $\mathbf{S}_{0i}$ . Away from half filling, the continuum limit of the Hamiltonian (3) proceeds in the same way as in the first model and with the same degree of approximations than in Eq. (2), one has:

$$\mathcal{H}_2 \simeq \frac{v_F}{2} \sum_{a=1}^2 \left( (\partial_x \Phi_{ac})^2 + (\partial_x \Theta_{ac})^2 \right) + \frac{2\pi v_H}{3} (\mathbf{J}_{0R}^2 + \mathbf{J}_{0L}^2) + \frac{2\pi v_F}{3} \sum_{a=1}^2 (\mathbf{J}_{aR}^2 + \mathbf{J}_{aL}^2) + g (\mathbf{J}_{0R} \cdot \mathbf{I}_L + \mathbf{J}_{0L} \cdot \mathbf{I}_R) \quad (4)$$

where  $\Phi_{ac}$  is a bosonic field ( $\Theta_{ac}$  being the dual field) associated with charge fluctuations in the  $a$ th channel. The chiral  $SU(2)_1$  currents  $\mathbf{J}_{aR,L}$  correspond to the chiral uniform part of the electron spin density whereas  $\mathbf{J}_{0R,L}$  are the chiral  $SU(2)_1$  currents of the local moments.

*Toulouse point solution.* The next step of the approach is to use the representation of two  $SU(2)_1$  currents in terms of four Majorana fermions  $\xi^0$  and  $\bar{\xi}$  [5,17]:

$\mathbf{I}_{R,L} = -i \vec{\xi}_{R,L} \wedge \vec{\xi}_{R,L}/2$ ,  $\mathbf{J}_{1R,L} - \mathbf{J}_{2R,L} = i \vec{\xi}_{R,L} \xi_{R,L}^0$ . The Hamiltonians (2,4) share then the same structure:  $\mathcal{H}_{1,2} = \mathcal{H}_c^0 + \mathcal{H}^0(\xi^0) + \bar{\mathcal{H}}$  where  $\mathcal{H}_c^0$  is the free Hamiltonian of the charge bosonic fields and  $\mathcal{H}^0(\xi^0)$  is the free Hamiltonian of the massless Majorana fermion  $\xi^0$ . All nontrivial physics is incorporated in  $\bar{\mathcal{H}}$  which separates into two commuting and chirally asymmetric parts:  $\bar{\mathcal{H}} = \bar{\mathcal{H}}_1 + \bar{\mathcal{H}}_2$ ,  $[\bar{\mathcal{H}}_1, \bar{\mathcal{H}}_2] = 0$  with

$$\bar{\mathcal{H}}_1 = \frac{\pi v_1}{2} \mathbf{I}_R^2 + \frac{2\pi v_0}{3} \mathbf{J}_{0L}^2 + g \mathbf{J}_{0L} \cdot \mathbf{I}_R \quad (5)$$

with  $v_1 = v_H$ ,  $v_0 = v_F$  for the 1DEG in a special active environment and the reverse for the two-channel KHL;  $\bar{\mathcal{H}}_2$  is obtained from  $\bar{\mathcal{H}}_1$  by inverting chiralities of all the currents. For an antiferromagnetic Kondo coupling ( $g > 0$ ), the interacting part of  $\bar{\mathcal{H}}$  is marginally relevant. Naively one will thus expect that the model will enter a massive strong coupling region. However, due to the chiral asymmetry of  $\bar{\mathcal{H}}$ , the effective interaction flows towards a conformally invariant intermediate fixed point with a smaller central charge than in the ultraviolet. Stated differently, the interaction produces a mass gap in some sectors but there are still some degrees of freedom that remain critical in the infrared (IR). The nature of the fixed point of chirally asymmetric current-current models with the structure (5) have been determined by Andrei et al. [14] by a combination of CFT and Bethe ansatz techniques. This fixed point belongs to the class of chirally stabilized fluids introduced in Ref. [14] describing the IR behavior of one-dimensional interacting chiral fermions. A simpler way to identify the critical degrees of freedom of model (5) in the IR limit can be done by means of a Toulouse point approach as in the two-channel Kondo model [3]. By allowing anisotropic interaction ( $g \rightarrow g_{\parallel}, g_{\perp}$ ), the  $U(1)$  version of the Hamiltonian  $\bar{\mathcal{H}}$  can be mapped onto free Majorana fermions for a special value of the interaction  $g_{\parallel}^*$ . Although the position of the solvable point is non-universal, the Toulouse limit solution captures the physical and universal properties of the models including the  $SU(2)$  case. The details of the approach can be found in Refs. [18,13] and we shall now briefly review it to fix the notations. The starting point of the Toulouse solution is the Abelian bosonization of all  $SU(2)_1$  currents  $\mathbf{J}_a, a = 0, 1, 2$  in terms of massless bosonic fields  $\varphi_a$  [5]:  $J_{aR,L}^z = (1/\sqrt{2\pi})\partial_x \varphi_{aR,L}$ ,  $J_{aR,L}^+ = (1/2\pi a_0)e^{\mp i\sqrt{8\pi}\varphi_{aR,L}}$ . Introducing the symmetric combination of the fields:  $\varphi_+ = (\varphi_1 + \varphi_2)/\sqrt{2}$ , the  $SU(2)_2$  current  $\mathbf{I}$  writes:  $I_{R,L}^z = (1/\sqrt{\pi})\partial_x \varphi_{+R,L}$ ,  $I_{R,L}^+ = (i/\sqrt{\pi a_0})\xi_{R,L}^3 \kappa e^{\mp i\sqrt{4\pi}\varphi_{+R,L}}$ . A fermionic zero-mode operator  $\kappa$  has been introduced to ensure the proper anticommutation relations. The following canonical transformation is then performed:

$$\begin{aligned} \varphi_0 &= \sqrt{2} \bar{\Phi}_2 - \bar{\Phi}_1, \quad \varphi_+ = \sqrt{2} \bar{\Phi}_1 - \bar{\Phi}_2 \\ \vartheta_0 &= \sqrt{2} \bar{\Theta}_2 + \bar{\Theta}_1, \quad \vartheta_+ = \sqrt{2} \bar{\Theta}_1 + \bar{\Theta}_2 \end{aligned} \quad (6)$$

where  $\vartheta_0$  and  $\vartheta_+$  (respectively  $\bar{\Theta}_1$  and  $\bar{\Theta}_2$ ) are the dual

fields associated with  $\varphi_0$  and  $\varphi_+$  (respectively  $\bar{\Phi}_1$  and  $\bar{\Phi}_2$ ). For a special positive value (Toulouse point) of  $g_{\parallel}$ , ( $g_{\parallel}^* = 4\pi(v_0 + v_1)/3$ ), the arguments of the interacting terms become those of free fermions so that they can be reformionized further with the introduction of a pair of Majorana fields ( $\eta, \zeta$ ) using the correspondence:  $\eta_{R,L} + i\zeta_{R,L} = (\kappa/\sqrt{\pi a_0}) e^{\pm i\sqrt{4\pi}\bar{\Phi}_{2R,L}}$ . One finally ends with:

$$\begin{aligned} \bar{\mathcal{H}} = & \frac{u_1}{2} [(\partial_x \bar{\Phi}_1)^2 + (\partial_x \bar{\Theta}_1)^2] - \frac{iu_2}{2} [\zeta_R \partial_x \zeta_R - \zeta_L \partial_x \zeta_L] \\ & - \frac{iv_1}{2} [\xi_R^3 \partial_x \xi_R^3 - \xi_L^3 \partial_x \xi_L^3] - \frac{iu_2}{2} [\eta_R \partial_x \eta_R - \eta_L \partial_x \eta_L] \\ & + im [\xi_R^3 \eta_L - \eta_R \xi_L^3] \end{aligned} \quad (7)$$

where  $m = g_{\perp}/2\pi a_0$ ,  $u_1 = (2v_1 - v_0)/3$ , and  $u_2 = (2v_0 - v_1)/3$ . The Toulouse solution is stable provided that all velocities are positive i.e.  $1/2 \leq v_0/v_1 \leq 2$ . The first two terms in Eq. (7) describe decoupled free massless bosonic and Majorana fields,  $\bar{\Phi}_1$  and  $\zeta$ , contributing to criticality with the central charge:  $c = 3/2$ . The remaining part of the Hamiltonian (7) has a spectral gap  $m$  and describes hybridization of the Majorana  $\xi^3$  and  $\eta$  fermions with different chiralities. Adding the contribution of the excitations that do not participate in the interaction, the total central charge in the IR of the 1DEG in a special active environment (respectively the two-channel KHL) is  $c = 3$  (respectively  $c = 4$ ). Apart from the charge degrees of freedom, the elementary excitations of the models in the IR are of different nature: The magnetic excitations are spinons defined as  $\sqrt{\pi/2}$  kinks of the bosonic field  $\bar{\Phi}_1$  describing an effective  $S=1/2$  Heisenberg chain and nonmagnetic singlet excitations associated with the two massless Majorana fermions  $\xi^0, \zeta$  (critical Ising degrees of freedom) referred as pseudocharge degrees of freedom in Refs. [18,13]. These  $Z_2$  excitations play a crucial role in the nontrivial IR physical properties of the models as we shall see now.

*Physical properties of the 1DEG in a special active environment.* The Green's function for the right-left moving fermions can be computed using the bosonic representation:  $(R, L)_{\sigma} = (\kappa_{\sigma}/\sqrt{2\pi a_0}) e^{i\tau_{\sigma}\pi/4} e^{\pm i\sqrt{4\pi}\varphi_{\sigma R,L}}$  where  $\tau_{\uparrow,\downarrow} = \pm 1$ ,  $\varphi_{\sigma R,L} = (\Phi_{cR,L} + \tau_{\sigma}\varphi_{0R,L})/\sqrt{2}$ , and  $\kappa_{\sigma}$  are Klein-factors to insure the correct anticommutation relations between fermion fields of different spin index [19]. Using the Toulouse basis (6), one then obtains at the IR fixed point the estimate:

$$\begin{aligned} \langle R_{\sigma}(x, \tau) R_{\sigma'}^{\dagger}(0, 0) \rangle \sim \\ \frac{\delta_{\sigma, \sigma'}}{(v_F \tau - ix)^{1/2} (u_1 \tau + ix)^{1/2} (u_2 \tau - ix)} \end{aligned} \quad (8)$$

which means that the system displays non-Fermi liquid properties; the left-moving Green's function is obtained from Eq. (8) by the substitution:  $i \rightarrow -i$ . The most interesting physical quantities are the correlation functions of the various possible order parameters. The spin-spin

correlation functions have been computed in Ref. [13] and the slowest ones are the staggered parts that decay as  $x^{-3/2}$ . Using the previous Abelian bosonization of the chiral fermions and the Toulouse basis (6), we find in the electronic sector the following representation for the charge density wave (CDW), spin density wave (SDW), singlet (SS) and triplet (ST) superconducting order parameters in terms of the different critical fields at the IR fixed point:

$$\begin{aligned} \mathcal{O}_{CDW} &= L_{\sigma}^{\dagger} R_{\sigma} \simeq e^{i\sqrt{2\pi}\Phi_c} \cos\left(\sqrt{2\pi}\bar{\Phi}_1\right) \zeta_L \zeta_R \\ \vec{\mathcal{O}}_{SDW} &= L_{\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} R_{\beta} \simeq i e^{i\sqrt{2\pi}\Phi_c} \zeta_R \zeta_L \\ &\quad \left[ \cos\left(\sqrt{2\pi}\bar{\Theta}_1\right), \sin\left(\sqrt{2\pi}\bar{\Theta}_1\right), \sin\left(\sqrt{2\pi}\bar{\Phi}_1\right) \right] \\ \mathcal{O}_{SS} &= -i L_{\alpha} (\sigma^y)_{\alpha\beta} R_{\beta} \simeq e^{-i\sqrt{2\pi}\Theta_c} \cos\left(\sqrt{2\pi}\bar{\Phi}_1\right) \zeta_R \zeta_L \\ \vec{\mathcal{O}}_{TS} &= -i L_{\alpha} (\vec{\sigma}\sigma^y)_{\alpha\beta} R_{\beta} \simeq i e^{-i\sqrt{2\pi}\Theta_c} \zeta_R \zeta_L \\ &\quad \left[ \cos\left(\sqrt{2\pi}\bar{\Theta}_1\right), -\sin\left(\sqrt{2\pi}\bar{\Theta}_1\right), \sin\left(\sqrt{2\pi}\bar{\Phi}_1\right) \right]. \end{aligned} \quad (9)$$

All these conventional order parameters have thus the same scaling dimension (2) and the corresponding pair correlation functions decay as  $x^{-4}$  i.e. much faster than in the one dimensional metals. One should notice that the  $Z_2$  pseudocharge excitations contribute in (9) through the density energy operator  $i\zeta_R \zeta_L$ . In a critical Ising model, there are other primary fields (order and disorder parameters) that have a smaller scaling dimension (1/8) which are highly *nonlocal* in terms of the original Majorana fermion [5]. This leads us to investigate the possibility that the ground state might be characterized by composite order parameters: staggered odd-frequency singlet pairing (c-SP) and staggered composite CDW (c-CDW). At the IR fixed point, we find the following correspondence for these order parameters:

$$\begin{aligned} \mathcal{O}_{c-SP} &= \vec{\mathcal{O}}_{TS} \cdot (\mathbf{n}_1 + \mathbf{n}_2) \sim e^{-i\sqrt{2\pi}\Theta_c} \mu_0 \mu_4 \\ \mathcal{O}_{c-CDW} &= \vec{\mathcal{O}}_{SDW} \cdot (\mathbf{n}_1 + \mathbf{n}_2) \sim e^{i\sqrt{2\pi}\Phi_c} \mu_0 \mu_4 \end{aligned} \quad (10)$$

where  $\mu_0, \mu_4$  (respectively  $\sigma_0, \sigma_4$ ) are the Ising disorder (respectively order) parameters associated with the Majorana fermions  $\xi^0, \zeta$ . From Eq. (10), we deduce the leading asymptotics of the correlation function corresponding to the composite order parameters:

$$\begin{aligned} \langle \mathcal{O}_{composite}(x, \tau) \mathcal{O}_{composite}^{\dagger}(0, 0) \rangle \sim \\ \frac{1}{(v_F^2 \tau^2 + x^2)^{1/2} (v_1^2 \tau^2 + x^2)^{1/8} (u_2^2 \tau^2 + x^2)^{1/8}} \end{aligned} \quad (11)$$

so that the composite order parameters induce the dominant instabilities and have enhanced long-range coherence. These fluctuations can be made even more enhanced upon switching on a very small ferromagnetic ( $g_{12} < 0$ ) exchange interaction between the spin chains:  $\mathcal{O}_{12} = g_{12} \mathbf{S}_1 \cdot \mathbf{S}_2$ . Indeed, at the IR fixed point, this perturbation mostly affects the pseudocharge sector leaving

intact the magnetic one:  $\mathcal{O}_{12} \simeq g_{12} i \xi_R^0 \xi_L^0$ . The Majorana fermion  $\xi^0$  acquires a positive mass i.e. its associated Ising model is in the disorder phase:  $\langle \mu_0 \rangle \neq 0$  and the  $Z_2$  symmetry with respect to the interchange of the two chains is now broken. All other degrees of freedom remain critical so that the pair correlation of the conventional order parameters (9) still decay as  $x^{-4}$  whereas from Eq. (10) one immediately observes that the composite ones decay now as  $x^{-5/4}$  instead of  $x^{-3/2}$  at  $g_{12} = 0$ .

*The two-channel KHL case.* The determination of the ground-state physical properties of the two-channel KHL proceeds in the same way as in the previous model. We first use the Abelian bosonisation of the right-left moving fermions:  $(R, L)_{a\sigma} = (\kappa_{a\sigma}/\sqrt{2\pi a_0}) e^{i\tau_\sigma \pi/4} e^{\pm i\sqrt{4\pi}\varphi_{a\sigma R,L}}$  where  $\varphi_{a\sigma R,L} = (\Phi_{acR,L} + \tau_\sigma \varphi_{aR,L})/\sqrt{2}$ , and  $\kappa_{a\sigma}$  are some Klein-factors [19]. The Green's functions of the chiral fermions have a similar structure than in Eq. (8) apart from they fall now with the distance with a power 5/4. The leading asymptotics of the correlation function of the localized spins have been computed in Ref. [13] and in particular the staggered part decays as  $x^{-3}$ . Using the Toulouse basis (6) and introducing the charge and relative charge bosonic fields  $\Phi_{\pm cR,L} = (\Phi_{1cR,L} \pm \Phi_{2cR,L})/\sqrt{2}$ , the order parameters for the electronic degrees of freedom can also be expressed in terms of the critical fields at the IR fixed point. We find the following correspondence for the conventional order parameters:

$$\begin{aligned} \mathcal{O}_{CDW} &= L_{a\sigma}^\dagger R_{a\sigma} \sim e^{i\sqrt{\pi}\Phi_{+c}} \cos\left(\sqrt{2\pi}\bar{\Phi}_1\right) \mathcal{I}_\Phi \\ \vec{\mathcal{O}}_{SDW} &= L_{a\alpha}^\dagger \vec{\sigma}_{\alpha\beta} R_{a\beta} \sim e^{i\sqrt{\pi}\Phi_{+c}} \\ &\left[ \cos\left(\sqrt{2\pi}\bar{\Theta}_1\right) \mathcal{I}_\Phi^\dagger, \sin\left(\sqrt{2\pi}\bar{\Theta}_1\right) \mathcal{I}_\Phi^\dagger, -\sin\left(\sqrt{2\pi}\bar{\Phi}_1\right) \mathcal{I}_\Phi \right] \\ \mathcal{O}_{SS} &= -iL_{a\alpha}(\sigma^y)_{\alpha\beta} R_{a\beta} \sim e^{-i\sqrt{\pi}\Theta_{+c}} \cos\left(\sqrt{2\pi}\bar{\Phi}_1\right) \mathcal{I}_\Theta^\dagger \\ \vec{\mathcal{O}}_{TS} &= -iL_{a\alpha}(\vec{\sigma}\sigma^y)_{\alpha\beta} R_{a\beta} \sim e^{-i\sqrt{\pi}\Theta_{+c}} \\ &\left[ -\cos\left(\sqrt{2\pi}\bar{\Theta}_1\right) \mathcal{I}_\Theta, \sin\left(\sqrt{2\pi}\bar{\Theta}_1\right) \mathcal{I}_\Theta, \sin\left(\sqrt{2\pi}\bar{\Phi}_1\right) \mathcal{I}_\Theta^\dagger \right] \end{aligned}$$

where  $\mathcal{I}_\Phi = \mu_0 \mu_4 \cos(\sqrt{\pi}\Phi_{-c}) + i\sigma_0 \sigma_4 \sin(\sqrt{\pi}\Phi_{-c})$  and  $\mathcal{I}_\Theta$  is obtained from  $\mathcal{I}_\Phi$  by replacing  $\Phi_{-c}$  by its dual field  $\Theta_{-c}$ . The scaling dimension of all these conventional order parameters is 5/4 and their corresponding pair correlation functions decay thus as  $x^{-5/2}$  i.e. have no enhanced long-range coherence. Notice, however, that this decay is much slower than in the 1DEG in a special active environment case. Similarly, we find the following description at the IR fixed point for the staggered composite order parameters:

$$\begin{aligned} \mathcal{O}_{c-SP} &= \vec{\mathcal{O}}_{TS} \cdot \mathbf{n}_0 \sim e^{-i\sqrt{\pi}\Theta_{+c}} \mathcal{I}_\Theta \\ \mathcal{O}_{c-CDW} &= \vec{\mathcal{O}}_{SDW} \cdot \mathbf{n}_0 \sim e^{i\sqrt{\pi}\Phi_{+c}} \\ &(\mu_0 \mu_4 \cos(\sqrt{\pi}\Phi_{-c}) + 3i\sigma_0 \sigma_4 \sin(\sqrt{\pi}\Phi_{-c})) \end{aligned} \quad (12)$$

so that the scaling dimension of these operators is 3/4 and the pair correlation function of the composite order

parameters is still given by Eq. (11). The dominant instabilities of the two-channel KHL are thus of a composite nature as in the 1DEG in a special active environment. Finally, one should note that the scaling dimensions of the physical observables of the two-channel KHL obtained by the Toulouse point solution coincide with those of the recent CFT approach [15]. In this respect, we stress that, in contrast with the latter approach, the Toulouse point solution unables us to take into account the inherent velocities anisotropy in the problem.

In summary, we have shown that the two generalizations of the one dimensional KHL considered in this letter exhibit a nontrivial non-Fermi liquid low-temperature phase belonging to the class of chirally stabilized fluids with strong enhanced staggered composite pairing correlations.

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